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APPROXIMATE EXPRESSIONS FOR THE MAGNETIC FIELD FROM AN ELECTRIC CURRENT SOURCE IN A LAYERED CONDUCTING MEDIUM

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Approximate Expressions for the Magnetic Field from an Electric Current Source in a Layered Conducting Medium

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Abstract

An electric current source comprising point horizontal electric dipoles is proposed. An approximate description of the magnetic field from such a source is given. The environments considered are two-layer (2 semi-infinite media, typically the air and the sea) and three-layer (2 semi-infinite layers separated by a conducting slab, typically the air, sea and sea-bed).

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Approximate Expressions for the Magnetic Field from an Electric Current Source in a Layered Conducting Medium

1. *Introduction*

Reference 1 describes an electric source mathematical model in which a set of electric source parameters can be deduced from a measured electric field matrix. The environment is modelled by two semi-infinite media (the air and the sea-bed, the former having an electrical conductivity of zero) separated by a conducting *slab of finite thickness* and *non-trivial* conductivity (the sea). The source is modelled as either a line of point current sources and sinks, or by a continuous distribution of these, the density distribution function being described by a polynomial. In either case, the total current must be zero. In the discrete case the amplitudes of the currents must sum to zero, whilst in the continuous case the integral of the distribution function must be identically zero.

The effectiveness of the point source model to describe the electric fields has been well shown. However, there is considerable interest in determining the magnetic fields from such a source.

The electric field is determined by (relatively) simple image theory. However, application to the magnetic field case is not as straightforward since the currents (which produce the field) are distributed through the volume of the conducting media.

In this report, the simpler case of just two semi-infinite media, with a common boundary (typically the air and an infinitely deep sea) is considered first. Then the more general three-layer case is considered. In both cases, the aim is to produce expressions for the magnetic field from a point dipole source rather than current source/sinks. Since the latter must sum to zero to be physically realizable, any such source of n current source/sinks can be represented by (n-1) dipoles.

2. *Two-Layered Medium*

Consider the geometry of Figure 1, in which layer 1 is infinitely deep ($d = \infty$), or alternatively $\sigma_1 = \sigma_2$. In practical terms, this is equivalent to very deep sea-water, where the source and sensor are not near the bottom. For a point horizontal dipole in the x-axis, of amplitude P, at (x_0, y_0, z_0), reference 2 gives the expected magnetic fields in layer 0 and layer 1 at the point (x, y, z). These are given in cylindrical coordinates as follows (for the case where $\sigma_0 = 0$):

$$B_{r_0} = \frac{-\mu_0 P \sin\phi}{4\pi}\left[\frac{(z-z_0)}{[r^2+(z-z_0)^2]^{3/2}} + \frac{1}{r^2}\left(\frac{(z-z_0)}{[r^2+(z-z_0)^2]^{1/2}} - 1\right)\right]$$

$$B_{\phi_0} = \frac{\mu_0 P \cos\phi}{4\pi}\left[\frac{1}{r^2}\left(\frac{(z-z_0)}{[r^2+(z-z_0)^2]^{1/2}} - 1\right)\right]$$

$$B_{z_0} = \frac{\mu_0 P \sin\phi}{4\pi}\left(\frac{r}{[r^2+(z-z_0)^2]^{3/2}}\right)$$

$$B_{r_1} = \frac{-\mu_0 P \sin\phi}{4\pi}\left[\frac{(z-z_0)}{[r^2+(z-z_0)^2]^{3/2}} - \frac{1}{r^2}\left(\frac{(z+z_0)}{[r^2+(z+z_0)^2]^{1/2}} + 1\right)\right]$$

$$B_{\phi_1} = \frac{-\mu_0 P \cos\phi}{4\pi}\left[\frac{(z-z_0)}{[r^2+(z-z_0)^2]^{3/2}} + \frac{(z+z_0)}{[r^2+(z+z_0)^2]^{3/2}} + \frac{1}{r^2}\left(\frac{(z+z_0)}{[r^2+(z+z_0)^2]^{1/2}} + 1\right)\right]$$

$$B_{z_1} = \frac{\mu_0 P \sin\phi}{4\pi}\left(\frac{r}{[r^2+(z-z_0)^2]^{3/2}}\right)$$

It is of particular interest that B is independent of either conductivity (σ_0, σ_1), except, of course, that the conductivity of the layer containing the source will determine the current flow (hence P), at least for a constant voltage source (as would be the case for a galvanic source such as the potential difference between a ship's propellor and hull).

To convert the above to Cartesian coordinates, the following apply:

$$B_x = B_r \cos\phi - B_\phi \sin\phi$$

$$B_y = B_r \sin\phi + B_\phi \cos\phi$$

and

$$\sin\phi = (y - y_0), \cos\phi = (x - x_0), r^2 = (x - x_0)^2 + (y - y_0)^2$$

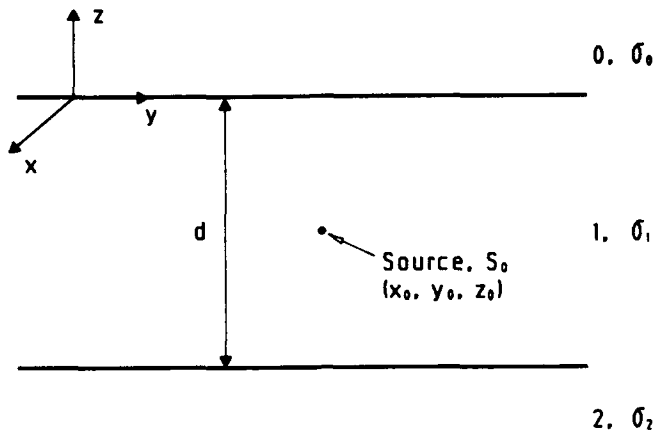

Figure 1: Geometry Definition

3. Three-Layered Medium

Now consider the full geometry of Figure 1. In this, the three layers have conductivities, σ_0, σ_1 and σ_2. Typically, for medium 0 being air, $\sigma_0 = 0$. A source, S_0 is located at (x_0, y_0, z_0) within medium 1, whose thickness is $d(d > 0)$.

In the first instance, this source is taken as a point source or sink of current, I, and is considered in isolation. Reference 3 gives the electric scalar potential, V, in the three media as follows:

$$V_0 = \frac{I}{4\pi\sigma_1}(1+Q_1)\left[\frac{1}{[r^2+(z-z_0)^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{Q_1^{k-1}Q_2^k}{[r^2+(z+2kd+z_0)^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{Q_1^k Q_2^k}{[r^2+(z+2kd-z_0)^2]^{1/2}}\right]$$

$$V_1 = \frac{I}{4\pi\sigma_1}\left[\frac{1}{[r^2+(z-z_0)^2]^{1/2}} + \frac{Q_1}{[r^2+(z+z_0)^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{Q_1^k Q_2^k}{[r^2+(z-2kd-z_0)^2]^{1/2}}\right.$$
$$\left. + \sum_{k=1}^{\infty}\frac{Q_1^{k+1} Q_2^k}{[r^2+(z-2kd+z_0)^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{Q_1^{k-1} Q_2^k}{[r^2+(z+2kd+z_0)^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{Q_1^k Q_2^k}{[r^2+(z+2kd-z_0)^2]^{1/2}}\right]$$

$$V_2 = \frac{I(1+Q_2)}{4\pi\sigma_1}\left[\frac{1}{[r^2+(z-z_0)^2]^{1/2}} + \frac{Q_1}{[r^2+(z+z_0)^2]^{1/2}}\right.$$
$$\left. + \sum_{k=1}^{\infty}\frac{Q_1^k Q_2^k}{[r^2+(z-2kd-z_0)^2]^{1/2}} + \sum_{k=1}^{\infty}\frac{Q_1^{k+1} Q_2^k}{[r^2+(z-2kd+z_0)^2]^{1/2}}\right]$$

where

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

$$Q_1 = (\sigma_1 - \sigma_0) / (\sigma_1 + \sigma_0), \quad Q_2 = (\sigma_1 - \sigma_2) / (\sigma_1 + \sigma_2)$$

Following reference 4, the magnetic field from this source is determined by invoking Ampere's law

$$\nabla \wedge \mathbf{B} = \mu_0 \sigma \mathbf{E}$$

Thus, by Stokes' theorem:

$$\int_S \nabla \wedge \mathbf{B} . d\mathbf{A} = \int \mathbf{B} . d\mathbf{l} = \mu_0 \sigma \int \mathbf{E} . d\mathbf{A}$$

This problem can be considered as one with cylindrical symmetry, with the z-axis being the axis of symmetry. Consider, then, the geometry of Figure 2, where the cable carrying current to this "monopole" source is explicitly included (the field from this is subtracted later).

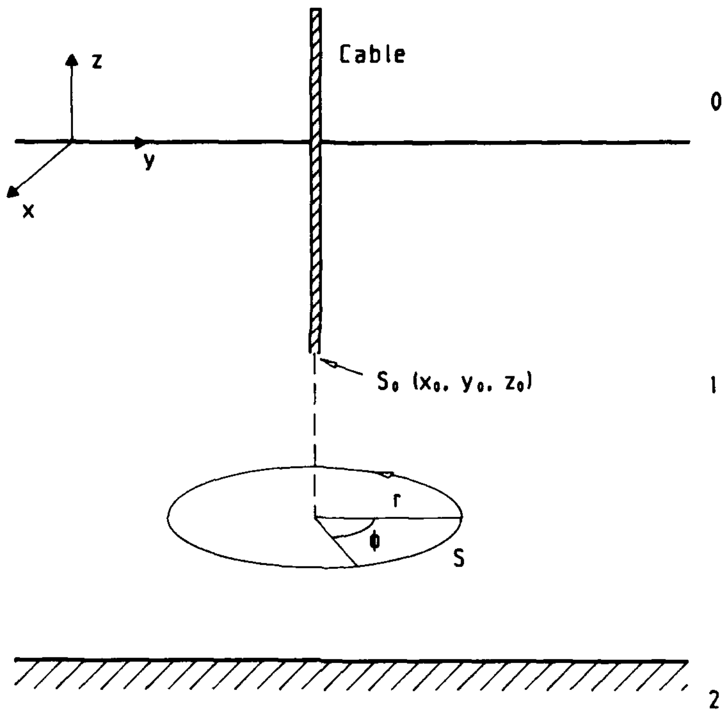

Figure 2: Cylindrical Symmetry Case

Since the z-axis is the axis of symmetry,

$$\int \mathbf{B}.\mathbf{dl} = B_{\phi}.2\pi r$$

Now

$$\mathbf{dA} = \mathbf{k}\, r dr\, d\phi$$

where k is the unit vector in the z-axis

$$\therefore \int \mathbf{E}.\mathbf{dA} = \int \mathbf{E}.\mathbf{k}\, r dr\, d\phi$$

and

$$\mathbf{E}.\mathbf{k} = E_z = E_z(r)$$

i.e. E_z is independent of ϕ

$$\therefore \int \mathbf{E.dA} = 2\pi \int E_z \, rdr$$

$$\therefore B_{\phi_n}.2\pi r = \mu_0 \sigma_n 2\pi \int_0^r E_z \, rdr$$

where n refers to the medium containing the field point (n = 0, 1, 2).

$$B_\phi = \frac{\mu_0 \sigma_n}{r} \int_0^r E_z \, rdr$$

Now, $E_z = - \partial V / \partial z$, and V is known. In general,

$$E_z = \sum_j E_{z_j}$$

where $E_z j$ is of the form:

$$\therefore \int E_{z_j} rdr = \frac{I}{4\pi\sigma_1} (z - z_j) \int_0^r \frac{rdr}{[r^2 + (z - z_j)^2]^{3/2}}$$

and the $\{j\}$ refer to the various source and image terms.

$$\int E_{z_j} rdr = \frac{-I}{4\pi\sigma} \left[\frac{(z - z_j)}{[r^2 + (z - z_j)^2]^{1/2}} \right]_D$$

$$= \frac{I}{4\pi\sigma} \left[sgn(z - z_j) - \frac{(z - z_j)}{[r^2 + (z - z_j)^2]^{1/2}} \right]$$

$$\therefore B_{\phi_{n_j}} = \frac{\mu_0 I}{4\pi r} \left[sgn(z - z_j) - \frac{(z - z_j)}{[r^2 + (z - z_j)^2]^{1/2}} \right]$$

Let $\sigma_0 = 0$ (air); $\therefore\ Q_1 = 1$ and Q_2 can be written as Q. For medium 0 (air) $z > 0,\ -d < z_0 < 0$.

Since $B_{\phi 0} \propto \sigma_n$, then $B_{\phi 0} = 0$.

However, the field from the supply cable must be subtracted from this $B_{\phi 0}$. For this medium, we choose the cable to run from $-\infty$ to z_0. The field from the cable in this instance is given by:

$$B_{\phi_{cable}} = \frac{\mu_0 I}{4\pi r}\left(1 - \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}}\right)$$

$$\therefore\ B_{\phi_0} = \frac{-\mu_0 I}{4\pi r}\left(1 - \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}}\right)$$

Note that the field in air is quite independent of water depth and conductivity.

Consider medium 2 (sea-bed) next, where $z < -d,\ -d < z_0 < 0$. Here,

$$sgn(z \pm z_0) = -1$$
$$sgn(z - 2kd \pm z_0) = -1,\ \forall\ k \geq 1$$

Hence, implicitly including the feed cable, $B_{\phi 2}$ is given by:

$$B_{\phi_2} = \frac{-\mu_0 I}{4\pi r}(1 - Q)\left[\left(1 + \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}}\right)\right.$$
$$+ \left(1 + \frac{(z + z_0)}{(r^2 + (z + z_0)^2)^{1/2}}\right) + \sum_{k=1}^{\infty} Q^k \left(1 + \frac{(z - 2kd - z_0)}{(r^2 + (z - 2kd - z)_0)^2)^{1/2}}\right)$$
$$\left. + \sum_{k=1}^{\infty} Q^k \left(1 + \frac{(z - 2kd + z_0)}{(r^2 + (z - 2kd + z_0)^2)^{1/2}}\right)\right]$$

In this medium, we choose the cable to run from $+\infty$ to z_0, for which the field is given by:

$$B_{\phi_{cable}} = \frac{-\mu_0 I}{4\pi r}\left[(1 + \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}}\right]$$

Thus, $B_{\phi 2}$ reduces to:

$$B_{\phi_2} = \frac{-\mu_0 I}{4\pi r}\left[1 - \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}} + (1 - Q)\sum_{k=0}^{\infty} Q^k \left(\frac{(z - 2kd - z_0)}{(r^2 + (z - 2kd - z_0)^2)^{1/2}} + \frac{(z - 2kd + z_0)^2)^{1/2}}{(r^2 + (z - 2kd + z_0)^2)^{1/2}}\right)\right]$$

In medium 1 (sea), $-d < z < 0$, $-d < z_0 < 0$.
Here,

$$\left.\begin{aligned} sgn\ (z + 2kd \pm z_0) &= +1 \\ sgn\ (z - 2kd \pm z_0) &= -1 \end{aligned}\right) \forall\ k \geq 1$$
$$sgn\ (z + z_0) = -1$$

The regions $z > z_0$ and $z < z_0$ must be considered separately. In particular, for $z > z_0$ the feed cable is assumed to run from $-\infty$ to z_0 and $sgn\ (z - z_0) = +1$. However, for $z < z_0$, the cable runs from $+\infty$ to z_0 and $sgn\ (z - z_0) = -1$. In both cases, $B_{\phi 1}$ reduces to:

$$B_{\phi_1} = \frac{-\mu_0 I}{4\pi r}\left[\left(1 + \frac{(z + z_0)}{(r^2 + (z + z_0)^2)^{1/2}}\right) + \sum_{k=1}^{\infty} \frac{Q^k (z - 2kd - z_0)}{(r^2 + (z - 2kd - z_0)^2)^{1/2}} + \sum_{k=1}^{\infty} \frac{Q^k (z - 2kd + z_0)}{(r^2 + (z - 2kd + z_0)^2)^{1/2}} + \sum_{k=1}^{\infty} \frac{Q^k (z + 2kd + z_0)}{(r^2 + (z + 2kd + z_0)^2)^{1/2}} + \sum_{k=1}^{\infty} \frac{Q^k (z + 2kd - z_0)}{(r^2 + (z + 2kd - z_0)^2)^{1/2}}\right]$$

Since the total current must sum to zero, there will always be more than one source/sink for this source type. Consequently, it is convenient to represent the source as a line of point dipoles as in reference 1. The magnetic field from one such point dipole, in the more useful Cartesian coordinates, is determined as follows.

In general, we may say that:

$$B_x = -B_\phi \sin\phi, \quad B_y = B_\phi \cos\phi$$

and

$$\sin\phi = (y - y_0)/r, \quad \cos\phi = (x - x_0)/r$$

Let there be monopoles of amplitude $+I$ at $(x + \delta x)$ and $-I$ at $(x - \delta x)$, then the field from such a dipole is given by:

$$B_{x,y_{\text{dipole}}} = \lim_{\delta x \to 0} \left(B_{x,y}(X + \delta x) - B_{x,y}(X - \delta x) \right)$$

where $X = x - x_0$, $Y = y - y_0$. Specifically,

$$\begin{aligned} B_x &= -(B_{\phi_+} - B_{\phi_-})\frac{y}{r} \\ B_y &= B_{\phi_+}\frac{(x+\delta x)}{r_+} - B_{\phi_-}\frac{(x-\delta x)}{r_-} \end{aligned}$$

where

$$\begin{aligned} r_\pm^2 &= (X \pm \delta x)^2 + Y^2 \\ B_{\phi_\pm} &= B_\phi(X \pm \delta x) \end{aligned}$$

Now, B_ϕ contains terms of the form:

$$\frac{A}{r^2}, \quad \frac{A z_j}{r^2 (r^2 + z_j^2)^{1/2}}$$

Let $R_j^2 = r^2 + z_j^2$. To first order,

$$\frac{1}{r_+^2} - \frac{1}{r_-^2} = -\frac{4x\delta x}{r^4}$$

$$\frac{1}{r_+^2(r_+^2+z_j^2)^{1/2}} - \frac{1}{r_-^2(r_-^2+z_j^2)^{1/2}} = \frac{1}{R_j}\left(\frac{2}{r^2} + \frac{1}{R_j^2}\right)$$

and

$$\frac{(X+\delta x)}{r_+^2} - \frac{(X-\delta x)}{r_-^2} = \frac{2\delta x}{r^2}\left(1 - 2\left(\frac{x}{r}\right)^2\right)$$

$$\frac{(X+\delta x)}{r_+^2(r_+^2+z_j^2)^{1/2}} - \frac{(X-\delta x)}{r_-^2(r_-^2+z_j^2)^{1/2}} = \frac{2\delta x}{r^2 R_j}\left[1 - 2\left(\frac{X}{r}\right)^2 - \left(\frac{X}{R_j}\right)^2\right]$$

The fields may now be determined. However, the field from the return path must be included. This is given by the familiar equation for a point electric dipole source in a homogeneous medium.

Specifically, the field from a current element flowing between $(X + \delta x)$ and $(X - \delta x)$ is given by

$$B_\theta = \frac{\mu_0 I}{4\pi\rho}\left(\frac{(X+\delta x)}{(\rho^2+(X+\delta x)^2)^{1/2}} - \frac{(X-\delta x)}{(\rho^2+(X-\delta x)^2)^{1/2}}\right)$$

where $\rho^2 = (y - y_0)^2 + (z - z_0)^2$.

θ is in the $(y - z)$ plane such that in $\sin\theta = (z - z_0)/\rho$, $\cos\theta = (y - y_0)/\rho$.
Expanding,

$$B_\theta = \frac{\mu_0 P}{4\pi\rho R}\left(1 - \left(\frac{X}{R}\right)^2\right)$$

where $P = 2I\delta x$, the dipole moment. Thus,

$$B_y = \frac{-\mu_0 P(z - z_0)}{4\pi R^3}, \qquad B_x = \frac{\mu_0 P(y - y_0)}{4\pi R^3}$$

Note that this return path represents the only source of vertical field and is independent of depths and conductivities.

Thus, the complete dipole fields may be determined as follows:

Medium 0

$$B_{x_0} = \frac{-\mu_0 P}{4\pi}\frac{X}{r}\frac{Y}{r}\left[\left(1 - \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}}\right)\frac{2}{r^2} - \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{3/2}}\right]$$

$$B_{y_0} = \frac{-\mu_0 P}{4\pi}\left[\left(1 - 2\left(\frac{X}{r}\right)^2\right)\left(1 - \frac{(z - z_0)}{(r^2 + (z - z_0)^2)^{1/2}}\right)\frac{1}{r^2} - \frac{(Z - z_0)}{(r^2 + (z - z_0)^2)^{3/2}}\left(1 - \left(\frac{X}{r}\right)^2\right)\right]$$

$$B_{z_0} = \frac{\mu_0 P}{4\pi}\frac{Y}{R^3}$$

Medium 1

$$B_{x_1} = \frac{-\mu_0 P}{4\pi}\frac{X}{r}\frac{Y}{r}\left[\left(1 + \frac{(z + z_0)}{(r^2 + (z + z_0)^2)^{1/2}}\right)\frac{2}{r^2} + \frac{(z + z_0)}{(r^2 + (z + z_0)^2)^{3/2}}\right.$$
$$\left. + \sum_{k=1}^{\infty} \frac{Q^k(z \pm 2kd \pm z_0)}{(r^2 + (z \pm 2kd \pm z_0)^2)^{1/2}}\left(\frac{2}{r^2} + \frac{1}{(r^2 + (z \pm 2kd \pm z_0)^2)}\right)\right]$$

where the ± should be read as "plus and minus", implying 4 summations in this case.

$$B_{y_1} = \frac{-\mu_0 P}{4\pi}\left[\left[\left(1-2\left(\frac{X}{r}\right)^2\right)\left(1+\frac{(z+z_0)}{(r^2+(z+z_0)^2)^{1/2}}\right)\frac{1}{r^2}\right.\right.$$

$$-\left(\frac{X}{r}\right)^2\frac{(z+z_0)}{(r^2+(z+z_0)^2)^{3/2}} - \frac{(z-z_0)}{(r^2+(z-z_0)^2)^{3/2}}$$

$$\left.\left. + \sum_{k=1}^{\infty}\frac{Q^k(z \pm 2kd \pm z_0)}{r^2(r^2+(z\pm 2kd \pm z_0)^2)^{1/2}}\left[1-2\left(\frac{X}{r}\right)^2 - \frac{X^2}{(r^2+(z\pm 2kd\pm z_0)^2)}\right]\right]\right]$$

$$B_{z_1} = \frac{\mu_0 P}{4\pi}\frac{Y}{R^3}$$

Medium 2

$$B_{x_2} = \frac{-\mu_0 P}{4\pi}\frac{X}{r}\frac{Y}{r}\left[\left[\left(1-\frac{(z-z_0)}{(r^2+(z-z_0)^2)^{1/2}}\right)\frac{2}{r^2} - \frac{(z-z_0)}{(r^2+(z-z_0)^2)^{3/2}}\right.\right.$$

$$\left.\left. + (1-Q)\sum_{k=0}^{\infty}\frac{Q^k(z-2kd\pm z_0)}{(r^2+(z-2kd\pm z_0)^2)^{1/2}}\left(\frac{2}{r^2}+\frac{1}{(r^2+(z-2kd\pm z_0)^2)}\right)\right]\right]$$

$$B_{y_2} = \frac{-\mu_0 P}{4\pi}\left[\frac{1}{r^2}\left(1-2\left(\frac{X}{r}\right)^2\right)\left(1-\frac{(z-z_0)}{(r^2+(z-z_0)^2)^{1/2}}\right)\right.$$

$$-\left(1-\left(\frac{X}{r}\right)^2\right)\frac{(z-z_0)}{(r^2+(z-z_0)^2)^{3/2}}$$

$$\left. + (1-Q)\sum_{k=0}^{\infty}\frac{Q^k(z-2kd\pm z_0)}{(r^2+(z-2kd\pm z_0)^2)^{1/2}}\left[1-2\left(\frac{X}{r}\right)^2 - \frac{X^2}{(r^2+(z-2kd\pm z_0)^2)}\right]\right]$$

$$B_{z_2} = \frac{\mu_0 P}{4\pi}\frac{Y}{R^3}$$

The above expressions can be readily checked by confirming the continuity of the components across the boundaries at $z = 0$ and $z = -d$. Further it is easy to show that for $Q \to 0$ or $d \to \infty$, the expressions reduce to those given earlier for the two layer case.

At $r = 0$ (directly above or below the source) the expressions as given are inappropriate. However, in such a case they may be reformulated as follows:

$$B_{x_{n|r=0}} = 0 \qquad \forall\, n = 0, 1, 2$$
$$B_{z_{n|r=0}} = 0 \qquad \forall\, n = 0, 1, 2$$

Now, $$\frac{z}{(r^2 + z^2)^{1/2}} = sgn\,(z)\left(1 - \frac{1}{2}\left(\frac{r}{z}\right)^2\right), \text{ to first order}$$

$$\frac{z}{(r^2 + z^2)^{3/2}} = \frac{sgn\,(z)}{z^2}\left(1 - \frac{3}{2}\left(\frac{r}{z}\right)^2\right)$$

The following are obtained:

$$B_{y_{0|r\to 0}} = \frac{\mu_0 P}{8\pi (z - z_0)^2}\left[1 - \frac{3}{4}\left(\frac{X}{z - z_0}\right)^2 - \frac{9}{4}\left(\frac{Y}{z - z_0}\right)^2\right]$$

$$\begin{aligned} B_{y_{1|r\to 0}} = -\frac{\mu_0 P}{8\pi}\Bigg[& \frac{1}{(z + z_0)^2}\left(1 - \frac{9}{4}\left(\frac{X}{z + z_0}\right)^2 - \frac{3}{4}\left(\frac{Y}{z + z_0}\right)^2\right) - \frac{2\, sgn\,(z - z_0)}{(z - z_0)^2} \\ & - \sum_{k=1}^{\infty} Q^k \Bigg[\frac{1}{(z + 2\,kd \pm z_0)^2}\left(1 - \frac{9}{4}\,\frac{X^2}{(z + 2kd \pm z_0)^2} - \frac{3}{4}\,\frac{Y^2}{(z + 2kd \pm z_0)^2}\right) \\ & - \frac{1}{(z - 2kd \pm z_0)^2}\left(1 - \frac{9}{4}\,\frac{X^2}{(z - 2kd \pm z_0)^2} - \frac{3}{4}\,\frac{Y^2}{(z - 2kd \pm z_0)^2}\right)\Bigg]\Bigg] \end{aligned}$$

$$\begin{aligned} B_{y_{2|r\to 0}} = \frac{-\mu_0 P}{8\pi}\Bigg[& \frac{1}{(z - z_0)^2}\left(1 - \frac{3}{4}\left(\frac{X}{z - z_0}\right)^2 - \frac{9}{4}\left(\frac{Y}{z - z_0}\right)^2\right) \\ & + (1 - Q)\sum_{k=0}^{\infty} Q^k \Bigg[\frac{1}{(z - 2kd \pm z_0)^2}\left(1 - \frac{9}{4}\,\frac{X^2}{(z - 2kd \pm z_0)^2} \right. \\ & \left. - \frac{3}{4}\,\frac{Y^2}{(z - 2kd \pm z_0)^2}\right)\Bigg]\Bigg] \end{aligned}$$

4. Comparison of Two-Layer and Three-Layer Solutions

The above expressions for the two-layer and three-layer case are readily programmed. Two sources are considered here as examples.

In the first, a horizontal dipole of 125 A.m (a current source/sink pair of 50 A separated by 2.5 m) is placed at 3.35 m depth in 21 m of water with a sea/sea-bed Q-factor of 0.8. The field is calculated at 20 m depth for the two-layer and three-layer case (directly below the source) and the results are shown in Figure 3. In this geometry, the fields are entirely athwartships.

Clearly the use of two-layer equations would produce a considerable over-estimate of the fields.

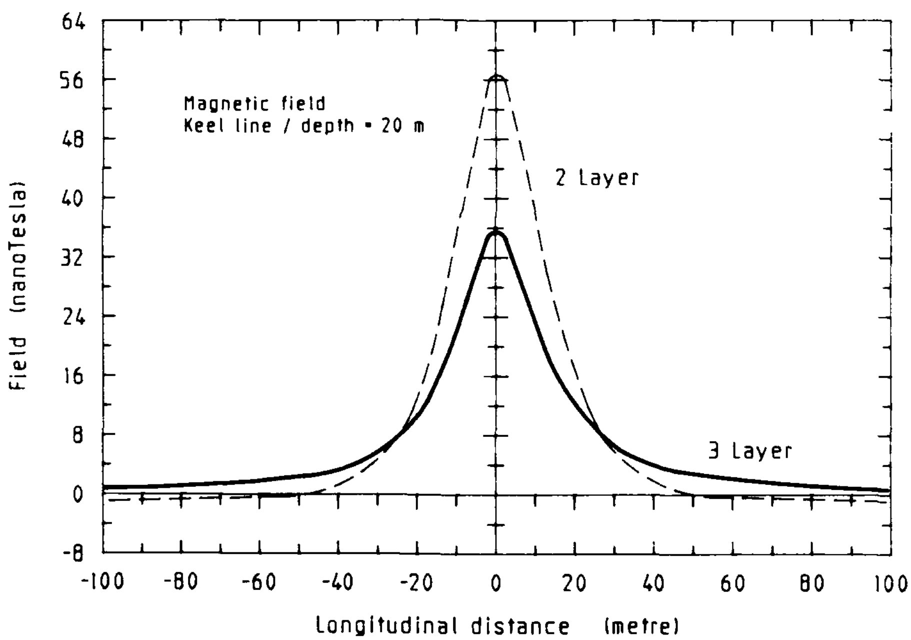

Figure 3: Below-water magnetic fields, simple source.

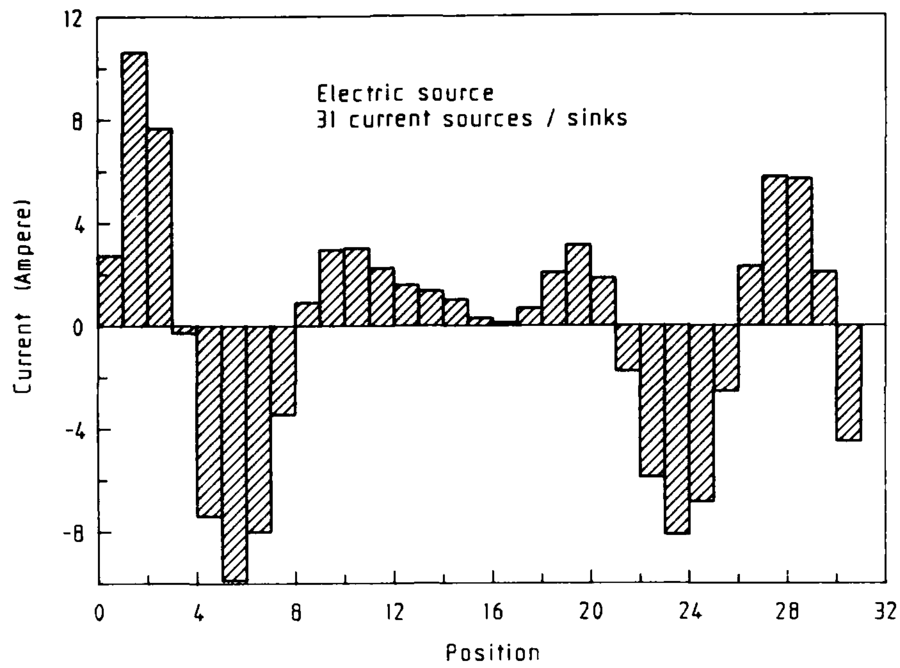

Figure 4: *Complex electric current source.*

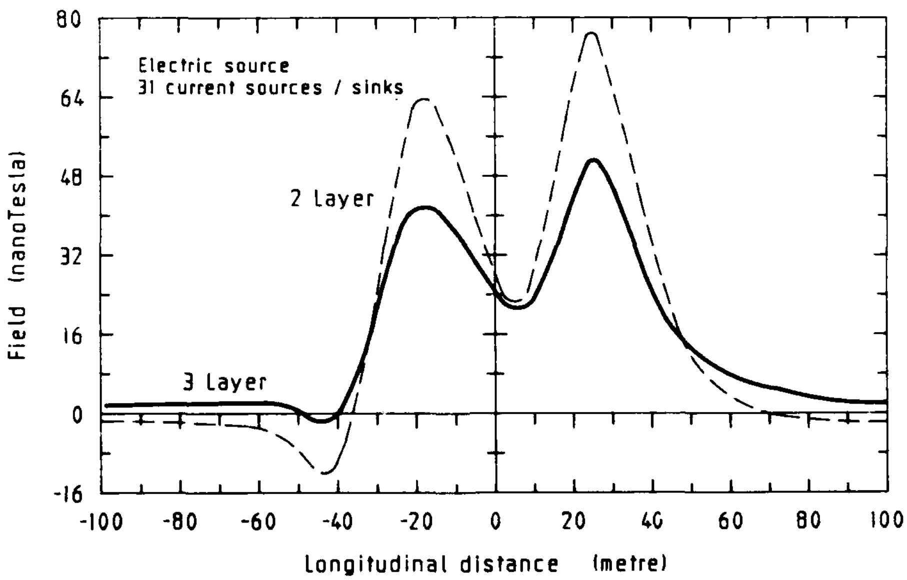

Figure 5: ***Below-water magnetic fields, complex source.***

In addition, the source defined in Figure 3 of Reference 1 (reproduced here as Figure 4) is also used, with the same geometric and environmental parameters. This source comprises 31 sources/sinks (where the total current sums to zero) separated by 2.67 m and has a far-field dipole moment of 100 A.m. The fields for this source, below water are given in Figure 5. Again the fields are less in the three-layer case at close range as would be expected.

5. Conclusion

The three-layer model developed herein illustrates the importance of explicitly including the conductivity of the sea-bed in calculations of the magnetic field from a horizontal electric dipole. Specifically, the fields could be up to 50% less than might otherwise be expected.

6. Acknowledgement

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7. References

1. Theobald, A.G. (1989).
   On the numerical modelling of an electrical source (ARE Technical Memo (UTM) 89120).

2. Kraichmann, M.B. (1970).
   Handbook of electromagnetic propagation in conducting media. White Oak, Silver Spring, MD: USN Ordnance Laboratory.

3. Jakosky, J.J. (1940).
   Exploration geophysics. Los Angeles: Times-Mirror Press.

4. Cooper, M.T. (1972).
   The underwater EM field of a submarine in the frequency range 0 to 0.3 Hz. (NCSL 109-72).

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ABSTRACT

An electric current source comprising point horizontal electric dipoles is proposed. An approximate description of the magnetic field from such a source is given. The environments considered are two-layer (2 semi-infinite media, typically the air and the sea) and three-layer (2 semi-infinite layers separated by a conducting slab, typically the air, sea and sea-bed).